GROMOV'S QUESTION

John Pardon – In high-school I spent a lot of my free time reading maths online and from books on my Dad's bookshelf. One day I came across a problem posed by a famous Russian mathematician, Gromov, working in Paris. This is the type of problem which is completely elementary to state, but seems to be expressing something fundamental which has yet to be understood. The mathematical theory of knots actually has its origins in physics. In the1800s, Lord Kelvin had a theory of atoms which proposed that different atoms were all small knots and the different knot types correspond to the different elements in the periodic table. Some years later a Scottish physicist, Tait, tabulated the simplest knots, starting with a knot, a Trefoil, and going on to more complicated knots. Mathematically, a knot is a closed loop in three-dimensional space. And the string that's tied up is infinitesimal, in particular you can tie infinitely many possible knots… If you take any particular physical piece of rope though, the number of knots you can tie is finite… This is something you can prove by pure thought without writing down any equations… It's essentially true, because the length of the rope is finite and has a definite thickness. And this is an example of how geometry, namely the length, and the thickness of the rope, can control the topology, what type of knot you can tie… Gromov's question was about another geometric quantity called distortion, and asked whether you can tie all knots with a fixed bound on the distortion…

Many years later, between junior and senior years of undergraduate, I spent the summer travelling in the UK, and when I was in Bath, I remember very clearly I was walking in the botanical gardens at Royal Victoria Park, and I realized there was a way to use distortion to control the topology of a knot. It's not so difficult to see that controlling the distortion controls the number of intersections between a knot and certain surfaces intersecting the knot. Something a little bit more subtle though, is that the number of intersections between the knot and certain surfaces in R3 can be used to control the topological type of the knot.

Walking in nature and exercising definitely stimulates the brain to come up with lots of new ideas, and in fact after I finished thinking about this problem on distortion, I became interested in another problem, about symmetries of three-dimensional space! And whether symmetric groups of three-dimensional space have to be continuous or whether they can be discontinuous, completely disconnected, discrete… And a key idea of how to address this problem came to me after going for a run near my home in North Carolina… Remember Tait who tabulated knots… So he made a lot of empirical observations about knots, from the knot tables, and these conjectures weren't proven until over a hundred years later in the 1990ties, when new knot invariance such as the Jones' polynomial were discovered… Moreover these conjectures, which had their origins in a physical theory which we now know to be completely wrong, were actually solved by new invariance, which were later found out to have a very close connection to modern physics, quantum field theory… And it sort of really stimulated a lot of new progress and interest in knot theory and the connection to physics, which at the moment we are only beginning to understand…

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