SPACE-TIME AND PRIME NUMBERS

Alain Connes – What I was attempting to do in my talk was... to excite the curiosity mostly of the young people who were there, and who might have been hesitant to devote their life to pure mathematics or even to the relations between maths and physics, by trying to explain how, precisely there are absolutely fundamental problems which are very far from being solved and which involve objects which can be perceived in an intuitive manner. The first is space-time which is relatively simple to perceive, and the second is the set of prime numbers, of course primes not just as a set but in the manner in which they fit inside numbers, inside real numbers. This is still very, very mysterious and in both cases, the case of space-time as well as the case of prime numbers, the fact that there are absolutely fundamental problems which are not yet resolved, neither in one case nor in the other. I mean in the case of space-time the fundamental problem which is well known but which deserves more detailed explanations is the problem of the compatibility between the two main theories: Einstein's gravitation and Quantum Mechanics.

Even concerning space, it took an enormous amount of time to understand that there is an underlying structure which is much more refined than what one would naively expect. Here of course you know the story with Galilée, Kepler, Newton, etc., and the understanding that the parabolas followed by an object one throws in the air are of the same family as the ellipses, that the trajectories of the planets are ellipses etc and that there are fascinating general laws which are far from obvious! What I mean is that, at the start, when one contemplates space, the Universe, etc, one does not at all get the feeling that it is structured. And this is the same for primes I mean for prime numbers. When one plays with prime numbers and computes the first ones, one does not perceive at all an underlying harmony. This very very deep underlying harmony was discovered by Euler, Riemann, Tchevychev, etc., and is the relation between precisely the distribution of prime numbers and the zeros of a certain function which is very very natural, analytic and possesses a great number of good properties; so what is absolutely striking in these two examples is that, in fact, these problems, of course, are interesting in themselves, per-se, but also because they generate an incredible number of ideas and that they slowly modify, each of them, and possibly in a convergent manner between the two problems, the notion of geometric space...But in all cases these problems have a great virtue, which is to create a momentum, a dynamics, towards a difficult goal.

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